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B. Sc. (NCF - NEP) (Sem.VI) Examination March - 2026

Mathematics Paper : XIV : Major -1 - MH - MJ-601

Group Theory - II

Time: 1 Hours] **[Total Marks: 25**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

Seat No.:

Student's Signature

Q. 1. Answer the following as directed: (Any FIVE) **05**

1. Let $G = (\mathbb{R}; +)$ and $\overline{G} = (\mathbb{R}^+; \bullet)$. Then find *Ker* ($\phi$); the *Kernel* of a homomorphism $\phi : G \to \overline{G}$ defined as $\phi\ (x) = 2^x; \forall x \in G$.
2. Let $\phi : G \to \overline{G}$ be a homomorphism of a group G into a group $\overline{G}$. Then prove that $\phi(e) = \overline{e}$ where e, $\overline{e}$ are the identity elements of groups G and $\overline{G}$ respectively.
3. If $\phi : G \to G$ is an automorphism of a finite group G, then prove that $o(\phi(a))|o(a)$; for every a in G.
4. Express the permutation (1 2 5) · (6 7 5 3) · (2 7) in S_7 as the product of disjoint cycles directly.
5. Determine whether the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 2 & 7 & 1 & 6 & 8 & 4 \end{pmatrix}$ in S_8 is even or odd.
6. Compute $\tau^{-1} \cdot \theta$; for the permutations $\tau = (1\ 5\ 4)$ and $\theta = (6\ 4\ 13) \cdot (2\ 6)$ in S_6.

Q. 2. Attempt any TWO: **10**

a. Prove that *Ker* (ϕ); the *Kernel* of a homomorphism $\phi : G \to \overline{G}$ a group G into a group $\overline{G}$; is a normal subgroup a group G.

b. Let $\phi : G \to \overline{G}$ be a homomorphism of a group G into a group $\overline{G}$; with the *Kernel*; *Ker* (ϕ). Prove that a homomorphism $\phi : G \to \overline{G}$ is an isomorphism if and only if *Ker* (ϕ) = (e); where e is the identity element of G.

c. Let $G = (a) = \{a^0 = e = a^4, a, a^2, a^3\}$ be a cyclic group of order 4. Then determine д(G); the group of all automorphisms of G.

Q. 3. **Attempt anyTWO:** **10**

a. (i) Verify whether $\tau^{-1} \cdot \omega \cdot \tau$ is even or odd permutation; for the permutations $\tau = (1\ 5\ 6)$ and $\omega = (1\ 3\ 4\ 2) \cdot (7\ 3)$ in S_7.

(ii) Find the Orbit of 5; for the permutation

$$\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 9 & 3 & 1 & 8 & 7 & 6 & 2 & 5 \end{pmatrix} \text{ in } S_9.$$

b. Prove that there is no permutation θ in S_7 satisfying $\theta^{-1} \cdot (7\ 1\ 3) \cdot \theta = (2\ 4) \cdot (3\ 6\ 5) \cdot$

c. Given the permutations; $\sigma = (2\ 1) \cdot (4\ 3)$ and $\tau = (5\ 4) \cdot (6\ 3)$ in S_6; find apermutation θ in S_6 satisfying $\theta^{-1} \cdot \sigma \cdot \theta = \tau \cdot$